

ESD RECORD COPY

RETURN TO
SCIENTIFIC & TECHNICAL INFORMATION DIVISION
(ESTI), BUILDING 1211

ESD ACCESSION LIST
ESTI Call No. AL 56214
Copy No. 1 of 1 cys.

Technical Note

1967-18

Navigation with High-Altitude Satellites: A Study of the Effects of Satellite-User Geometry on Position Accuracy

Carole D. Sullivan

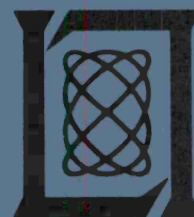
24 February 1967

Prepared under Electronic Systems Division Contract AF 19(628)-5167 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts



ES44

AD0651863

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the U.S. Air Force under Contract AF 19(628)-5167.

This report may be reproduced to satisfy needs of U.S. Government agencies.

Distribution of this document is unlimited.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

NAVIGATION WITH HIGH-ALTITUDE SATELLITES:
A STUDY OF THE EFFECTS OF SATELLITE-USER GEOMETRY
ON POSITION ACCURACY

CAROLE D. SULLIVAN

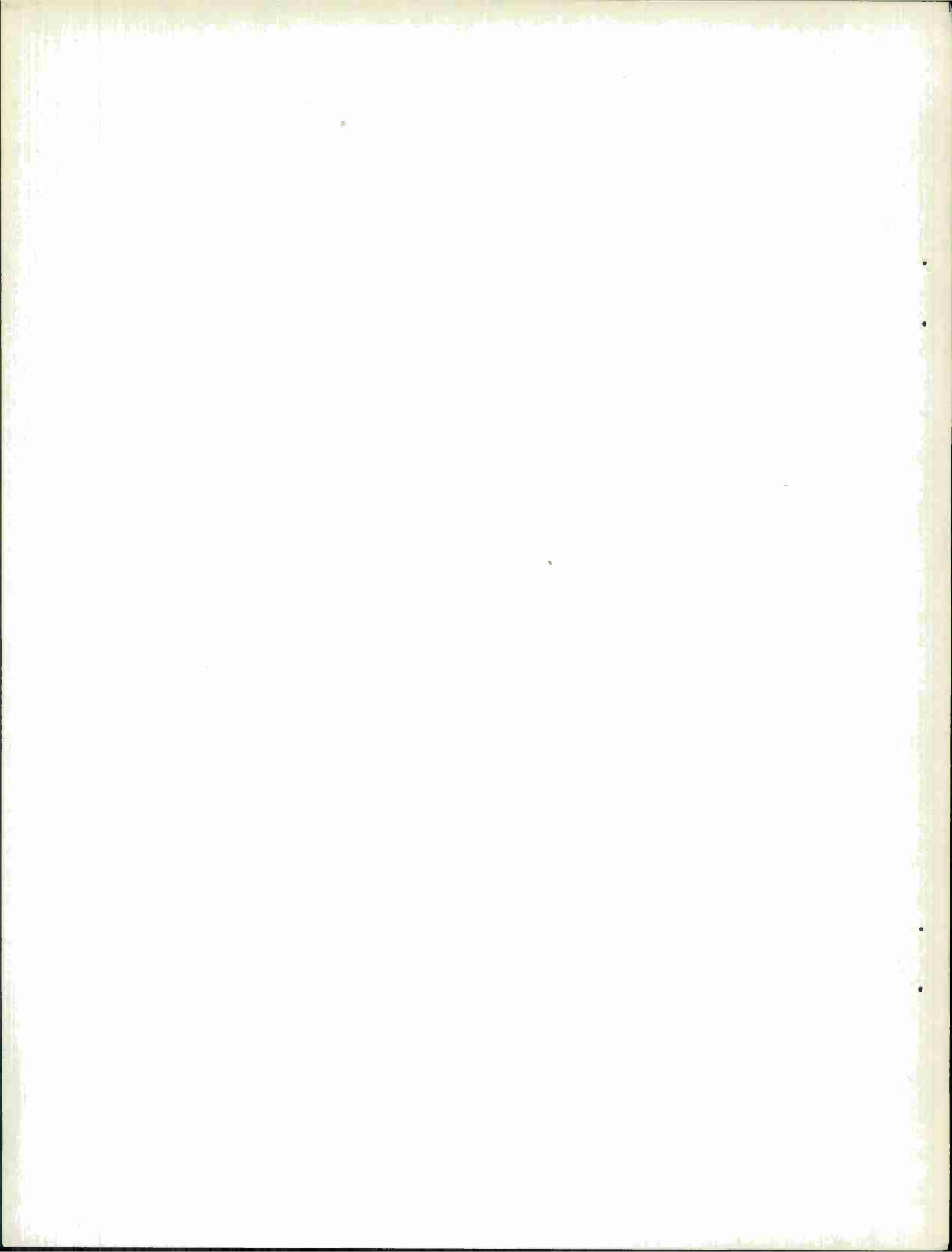
Group 66

TECHNICAL NOTE 1967-18

24 FEBRUARY 1967

LEXINGTON

MASSACHUSETTS



ABSTRACT

An error analysis of an hyperbolic navigation system using high-altitude satellites revealed that certain satellite arrangements result in long, narrow corridors on the earth (singular regions), in which navigation errors are very large. The study showed that in these singular regions the satellite-user geometry results in navigation equations which are sensitive to measurement errors and thus cannot be solved accurately for all three user coordinates.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

CONTENTS

Abstract	iii
I. Introduction	1
II. System Concept	1
III. Error Analysis	1
IV. Angle Between Hyperbolas	4
V. Results	7
VI. Summary	8

NAVIGATION WITH HIGH-ALTITUDE SATELLITES:
A STUDY OF THE EFFECTS OF SATELLITE-USER GEOMETRY
ON POSITION ACCURACY

I. INTRODUCTION

It has become apparent that high-altitude satellites have possibilities for navigation as well as communication. Although use of such satellites can allow the user to obtain an accurate position fix, Schweppe* pointed out that, for certain arrangements of the satellites and the user, the error in the user's determination of his position in three coordinates is extremely large. It is the purpose of this report to determine the extent of such singular regions, i.e., regions in which a user cannot make a unique three-coordinate fix, and to explore the reasons for their existence.

II. SYSTEM CONCEPT

The navigation system considered is an hyperbolic system in which a user receives timing signals transmitted from each of three synchronous-altitude satellites. The satellite clocks, from which the timing signals are derived, are assumed to be perfectly synchronized to a master clock. The user measures his height as well as the time of arrival of each of the satellite signals. In the noiseless case (that is, the user has made all his measurements perfectly and knows the satellite positions exactly), by using the difference in the time of arrival of the timing signals emitted by two of these satellites, the user can position himself on the locus of points satisfying this condition, that is, on an hyperbola of revolution. By taking one of these satellites and a third, he can determine another hyperboloid. The intersection of these two figures is not sufficient to determine the user's position in three coordinates. The user knows, however, he is on the surface of a sphere with radius equal to the radius of the earth plus his measured height. The intersection of the sphere and the hyperboloids will determine his position in three coordinates.

In practice, the user is not able to make perfect measurements and, therefore, the effects of measurement errors must be considered.

III. ERROR ANALYSIS

The quantities observed by the user are (a) the time of arrival of timing signals from each of the three satellites (as indicated on the user's clock) and (b) the user's height or distance from the center of the earth. Denote the user's position in three-dimensional space (origin at the center of the earth) by \underline{p}_u , a three-dimensional column vector (Fig. 1). Similarly, denote the satellite positions by the vectors \underline{p}_1 , \underline{p}_2 , \underline{p}_3 . If the user's clock were synchronized to the master

* F. C. Schweppe (private communication).

clock governing the satellite transmissions and there were no errors in the user's measurements, the time of arrival of a signal from a satellite would provide the user with the distance between himself and the satellite. That is, the user would observe indirectly

$$s_j = |\underline{p}_u - \underline{p}_j|, \quad j = 1, 2, 3$$

If the user clock were offset from the master clock by an unknown constant τ_o , the user would really observe

$$|\underline{p}_u - \underline{p}_j| + \tau_o = s_j + \tau_o, \quad j = 1, 2, 3$$

where τ_o has the dimension of length, since it has the effect of changing the apparent distance between the user and the satellite. The user height is

$$h_u = |\underline{p}_u|$$

and in the absence of any noise, the observables are

$$\underline{y} = \begin{bmatrix} s_1 + \tau_o \\ s_2 + \tau_o \\ s_3 + \tau_o \\ h_u \end{bmatrix}. \quad (1)$$

The quantities that \underline{y} depends on then are \underline{p}_u , \underline{p}_1 , \underline{p}_2 , \underline{p}_3 , and τ_o . We express these quantities as a column vector, called the state vector \underline{x} ,

$$\underline{x} = \begin{bmatrix} \underline{p}_u \\ \underline{p}_1 \\ \underline{p}_2 \\ \underline{p}_3 \\ \tau_o \end{bmatrix} \quad (2)$$

so that, as long as there is no noise,

$$\underline{y} = \underline{h}(\underline{x}) = \begin{bmatrix} h_1(\underline{x}) \\ h_2(\underline{x}) \\ h_3(\underline{x}) \\ h_4(\underline{x}) \end{bmatrix} = \begin{bmatrix} |\underline{p}_u - \underline{p}_1| + \tau_o \\ |\underline{p}_u - \underline{p}_2| + \tau_o \\ |\underline{p}_u - \underline{p}_3| + \tau_o \\ |\underline{p}_u| \end{bmatrix}. \quad (3)$$

The observables are, however, obscured by additive noise which can be expressed in vector form as

$$\underline{r} = \underline{y} + \underline{n} = \underline{h}(\underline{x}) + \underline{n}. \quad (4)$$

We assume that the a priori knowledge of the components of the state vector is in the form of a probability density. In particular, we take \underline{x} to have statistically independent Gaussian components with mean

$$E[\underline{x}] = \underline{x}_m$$

and covariance

$$W = E[(\underline{x} - \underline{x}_m)(\underline{x} - \underline{x}_m)']$$

where ()' denotes matrix transpose and E denotes the statistical expectation.* The noise vector \underline{n} is also taken to be Gaussian with zero mean. Under these assumptions, the problem of calculating the user position from noisy observables can be viewed as forming a statistical estimate of the state vector \underline{x} . In particular, the maximum a posteriori probability estimate of the state vector is formed, i.e., the estimate of \underline{x} that maximizes $p(\underline{x}|\underline{r})$. Let $\hat{\underline{x}}$ denote this estimate. The error analysis of such a navigation scheme consists of calculating the covariance matrix of the error vector $\underline{e} = (\underline{x} - \hat{\underline{x}})$.

In order to do this, the vector function $\underline{h}(\underline{x})$ is first linearized about the user's true position to make this a linear estimation problem. Let the matrix H denote the linear transformation required to obtain the linearized version of the general nonlinear function $\underline{h}(\cdot)$, i.e.,

$$\underline{h}(\underline{x}) \approx \underline{h}(\underline{x}_0) + H(\underline{x} - \underline{x}_0) \quad (5)$$

where

$$H = (h_{ij}) = \left(\frac{\partial h_i(\underline{x})}{\partial x_j} \right)_{(\underline{x}=\underline{x}_0)}$$

This linear approximation of the nonlinear function $\underline{h}(\cdot)$ is accurate when the difference between the estimate $\hat{\underline{x}}$ and \underline{x}_0 is small, as it should be for an accurate navigation system.

The resulting error covariance matrix

$$V = (v_{ij}) = E[\underline{e}\underline{e}']$$

has been calculated as[†]

$$V = W - WH[N + HWH']^{-1}HW = [H'N^{-1}H + W^{-1}]^{-1} \quad (6)$$

where W is as previously defined and N is the covariance matrix of the noise vector, i.e.,

$$N = (n_{ij}) = E[\underline{n}\underline{n}']$$

where n_k is an element of the noise vector \underline{n} .

Since we are interested in the estimate of \underline{p}_u , only a submatrix of V is pertinent to the actual navigation errors. The upper left 3×3 submatrix of V is the error covariance matrix of the components of the position estimate $\hat{\underline{p}}_u$. As our measure of the accuracy of navigation, we adopt the root-mean-squared (RMS) error between the estimate $\hat{\underline{p}}_u$ and the true position \underline{p}_u .

* The statistical expectation of a matrix is taken to be the matrix of expected values of each of the elements.

†D. L. Snyder, "Navigation with High-Altitude Satellites: A Study of the Errors in Position Determination," Technical Note 1967-11, Lincoln Laboratory, M.I.T. (6 February 1967), DDC 648828, H-803.

A computer program was written to calculate the V matrix and from it the RMS position error for any positions of the satellites and the user. The input to the program consists of the a priori standard deviation of the components of the state vector \underline{x} and the observation noise \underline{n} . These are taken to be

Ranging error	10 meters (RMS)
Height error	10 meters (RMS)
User position error	10^6 meters (RMS)
Satellite position error	10^2 meters (RMS)
Clock error	10^6 meters (RMS)

where the square of the ranging error and the square of the height error are the diagonal elements of the covariance matrix N , the other elements of this matrix being zero, since the observation errors are taken to be statistically independent. The squares of the remaining three parameters are the diagonal elements of the W matrix, which is also diagonal for the same reason. In order to eliminate any a priori knowledge of user position and clock error, the variances of these quantities should approach infinity. This limit was well approximated by using 10^6 meters as the a priori variance of these quantities, which proved to be of inestimable value in view of the finite word length of the digital computer used for the calculations. The other parameters lead to reasonable navigation accuracies and are realizable (hopefully) with practical equipment.* The program calculates the RMS error between the maximum a posteriori estimate of \underline{p}_u and the actual user's position for any user-satellite geometries.

Consider the case where the satellite positions are as follows:

	<u>Latitude</u>	<u>Longitude</u>
Satellite 1	0.0	30.0
Satellite 2	0.0	0.0
Satellite 3	0.0	-30.0

that is, when all three satellites are along the equator. The RMS errors for the user at various positions between 60° latitude and the equator, and 40° longitude and 0° longitude are given in Fig. 2. At 60° latitude, the RMS error is of the order of 10^2 meters for all longitudes and increases steadily as the user approaches the equator; at the equator, it has a value of 10^6 meters, but this is the a priori assumption of the position error (Fig. 3). The observations and calculations have not given the user any information about his position. This is a singular region of the type discovered by Schweppe.

We have seen that the RMS error varies markedly over the different user positions. The question then arises as to why certain arrangements of satellites and user cause large errors in position determination.

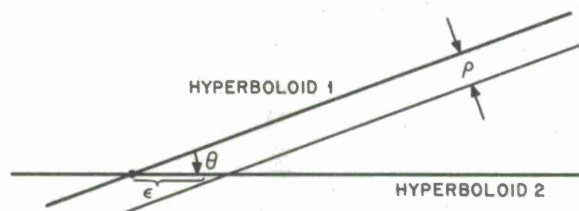
IV. ANGLE BETWEEN HYPERBOLAS

Consider again how a user would determine his position if this were a deterministic case (i.e., no noise and no errors in satellite position knowledge). He would calculate two hyperbolas of revolution and a sphere, all three of which he knows he must be on (Fig. 4). The intersection of these figures, therefore, provides his position fix. Suppose, however, instead of being a

* T. J. Goblick, Jr., "Navigation with High-Altitude Satellites: A Study of Ranging Errors," Technical Note 1966-46, Lincoln Laboratory, M.I.T. (26 August 1966), DDC 643851, H-752.

deterministic case, there is some error in the user's measurements and thus an error ρ in his calculations of the position of one hyperbola. This error would produce an error in position determination

$$\epsilon = \frac{\rho}{\sin \theta} \quad (7)$$



where θ denotes the angle formed by the hyperboloids intersected with the plane of the earth at the point of the user. If θ is close to 90° , $\sin \theta$ is close to one and the error ϵ in the user's determination of his position is nearly equal to the error ρ in his calculations. However, as $\theta \rightarrow 0$, $\sin \theta \rightarrow 0$ and $\epsilon \rightarrow \infty$, which indicates that the error ρ would be greatly magnified for small angles. The hyperboloids and the sphere intersect in such a way that the user, unable to measure perfectly, cannot determine his position accurately in three coordinates. The error in his position determination would be entirely dependent on his a priori knowledge.

To calculate θ , let \underline{p}_u be a three-dimensional vector (as in Sec. III), origin at the center of the earth, denoting the user's position with coordinates (x_u, y_u, z_u) , and let \underline{p}_1 , \underline{p}_2 , and \underline{p}_3 represent the three satellites with coordinates (x_1, y_1, z_1) , (x_2, y_2, z_2) , and (x_3, y_3, z_3) , respectively. Then the distance between the user and each of the satellites is

$$|\underline{p}_u - \underline{p}_i| = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} = s_i, \quad i = 1, 2, 3 \quad (8)$$

Since the time required for the user to receive the timing signals is a function of the distance between the user and the satellite, we can describe the hyperboloid on which the user is located as the locus of points such that the difference in the distance between that point and two other points is constant. One hyperboloid is determined by satellite 1, satellite 2, and $|s_1 - s_2|$; and another by satellite 2, satellite 3, and $|s_2 - s_3|$. From Eq. (8), the two hyperboloids are

$$f_1(x, y, z) = |s_1 - s_2| = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} - \sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2} \quad (9)$$

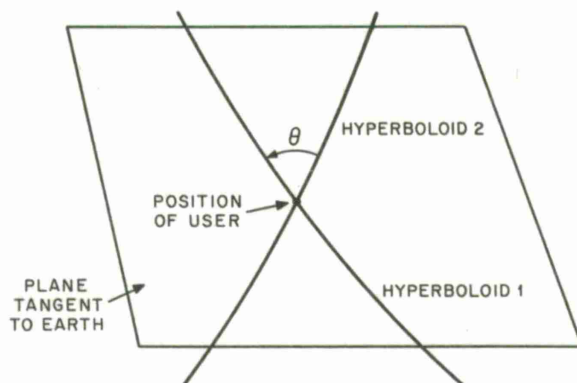
$$f_2(x, y, z) = |s_2 - s_3| = \sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2} - \sqrt{(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2} \quad (10)$$

If f is a function of three independent variables (x, y, z) , the gradient of the function is defined as

$$\nabla f = \frac{\partial f}{\partial x} \underline{i} + \frac{\partial f}{\partial y} \underline{j} + \frac{\partial f}{\partial z} \underline{k}$$

where \underline{i} , \underline{j} , and \underline{k} are unit vectors along the positive x , y , and z axes, respectively. Geometrically, ∇f , evaluated at the point (x_o, y_o, z_o) , is a vector whose direction is normal to the level surface [i.e., the set of all points such that $f(x, y, z) = c$] at the point (x_o, y_o, z_o) . In particular, $f_1(x, y, z)$ in Eq. (9) defines a family of hyperboloids, the value of $|s_1 - s_2|$ defining the particular hyperboloid. The gradient of f_1 evaluated at a point (x_o, y_o, z_o) will be a vector whose direction is normal to the particular hyperboloid passing through (x_o, y_o, z_o) . Likewise, we could find a vector whose direction is normal to the hyperboloid defined by $f_2(x, y, z)$ in Eq. (10) passing through the point (x_o, y_o, z_o) .

If ∇f_1 and ∇f_2 are the gradients of the functions f_1 and f_2 , then the angle Θ , formed by the intersections of the hyperboloids and the tangent plane of the sphere, can be found by computing the angle projected in the tangent plane between ∇f_1 and ∇f_2 evaluated at the user's position.



Let \underline{E} be a unit vector normal to the plane tangent to the sphere at the point of the user. Then

$$\underline{u}_1 = \frac{\nabla f_1 \times \underline{E}}{|\nabla f_1 \times \underline{E}|} \quad (11)$$

and

$$\underline{u}_2 = \frac{\nabla f_2 \times \underline{E}}{|\nabla f_2 \times \underline{E}|} \quad (12)$$

are unit vectors in the plane tangent to the sphere, and

$$\cos \Theta = \underline{u}_1 \cdot \underline{u}_2$$

or

$$\Theta = \cos^{-1}(\underline{u}_1 \cdot \underline{u}_2) \quad (13)$$

When Eq. (13) is evaluated for satellite-user geometries where navigation accuracy is known to be poor, the values of Θ should be near or exactly zero. In particular, if this expression is evaluated for the satellite geometry used in Sec. III at the user positions that had the maximum RMS position error, that is, when the user is located at any point along the equator, $\cos \Theta$ does equal one or the angle between the hyperbolas in the plane tangent to the earth equals zero (see Fig. 5).

Since it is more difficult to solve this equation for non-equatorial cases, a computer program was written. Input to the program consists of the longitude and latitude of the three

satellites and the user; $1/\sin \Theta$ is the output quantity rather than Θ , since the error magnification is seen in Eq. (7) to depend on $1/\sin \Theta$.

There is a direct correspondence between the user locations resulting in large values of $1/\sin \Theta$ and user locations with large RMS position errors for the same satellite geometry. The expression $1/\sin \Theta$ had a minimum value at 60° latitude for all longitudes and increased steadily until, along the equator, Θ became zero and $1/\sin \Theta$ could not be computed. It was also at 60° latitude that the minimum RMS position error occurred, and along the equator that the maximum occurred.

V. RESULTS

It is interesting to explore satellite-user geometries with less symmetry than the equatorial case of Fig. 5. The two computer programs were run, therefore, for other satellite and user positions.

The following satellite positions provide a case with only east-west symmetry.

	<u>Latitude</u>	<u>Longitude</u>
Satellite 1	0.0	30.0
Satellite 2	10.0	0.0
Satellite 3	0.0	-30.0

Navigation accuracy for user positions from $+60^\circ$ to -60° latitude and from -40° to $+40^\circ$ longitude was studied. A singular region again appeared. The maximum RMS error of the user at -40° longitude occurred at latitude $+41^\circ$; at 0° longitude, it appeared at 49° latitude; and at 40° longitude, the maximum was at 41° latitude. The effect of moving the satellite north was to move the singular region in that direction while the southern hemisphere became free from any singular points. The actual values of the errors are meaningful only in a relative sense because the region was not sampled finely enough to conclude that the worst point was found. The results of the program to calculate the angle between the hyperboloids for this same geometry again showed the maximum value of $1/\sin \Theta$ occurring at exactly the same user positions as the maximum RMS position errors [see Fig. 6(a-b)].

If satellite 2 is now moved to 15° latitude, 15° longitude, all symmetry is destroyed, and user positions from -40° to $+40^\circ$ longitude and $+75^\circ$ to -75° latitude must be studied. (The increase in the range of latitude is needed because of the more northerly position of the satellite.) Again, there is a narrow locus of singular points with the worst point this time occurring (user longitude 0°) at user latitude 68° . This, however, is nearly out of the region of mutual visibility* of all three satellites [see Fig. 7(a-b)]. The region of singular points could probably be moved entirely out of the region where the user could see all three satellites by further adjustment of the northern satellite.

We can conclude that the region of singular points exists whether the satellites are arranged symmetrically or unsymmetrically. The arrangement of the satellites does, however, have an effect on the location of the region of large errors.

* In this report, visibility curves are computed using an elevation angle of 7° .

Taking the positions of the satellites to be

	<u>Latitude</u>	<u>Longitude</u>
Satellite 1	0.0	45.0
Satellite 2	0.0	0.0
Satellite 3	0.0	-45.0

we find that the singular points again occur on the equator as expected, and that the maximum values of the RMS position errors are the same along the equator as for the previous equatorial case because of the a priori assumptions. If, however, we compare the RMS values of the user positions off the equator for the 30° and 45° cases, we find that the corresponding values for the 30° case are 2.1 times larger than those for the 45° case, and that $1/\sin \theta$ is 0.5 times larger for the 30° case [see Fig. 8(a-b)]. The effect of having the equatorial satellites 90° instead of 60° apart was a decrease in the width of the region of large errors. However, the region of mutual visibility when the two satellites are 90° apart is so limited that three satellites would not be sufficient to cover the North Atlantic, for example; whereas, this could be done if the satellites were only 60° apart.

Thus, if high-altitude satellites are to be used for navigation, consideration must be given to arranging the satellites so that the user will not be in a singular region, that is, so that the user will be able to determine his position accurately. But consideration must also be given to making the area of mutual visibility of the user and the satellites large enough for the system to be practical.

VI. SUMMARY

From the preceding, the following conclusions can be drawn:

- (1) It appears that all satellite geometries result in long, narrow corridors in which navigation accuracy is poor.
- (2) The cause of this singular region is geometric in nature; that is, the two hyperboloids and the sphere which locate the user intersect in such a way as to greatly magnify any error in the user's measurements.
- (3) A system with all satellites on the equator always yields a singular region along the equator. Thus a practical system would utilize at least one non-equatorial satellite.
- (4) If two of the satellites are on the equator and the third is located far enough north, the singular region can be shifted outside the range of mutual visibility.
- (5) If two of the satellites are on the equator, the singular region is located in the same hemisphere as the third satellite; i.e., the opposite hemisphere will be free from singular points.

ACKNOWLEDGMENT

The author wishes to thank Thomas J. Goblick, Jr., for his valuable assistance in the preparation of this report and Barney Reiffen for his very helpful comments.

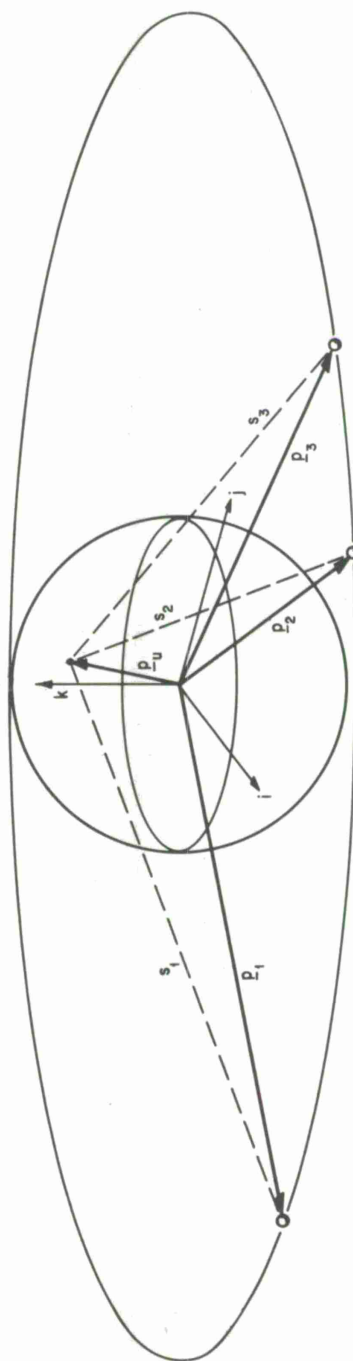


Fig. 1. Hyperbolic navigation system.

USER POSITION
 LAT LONG

	0.0	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000
60.0	9.3210 02	9.3190 02	9.3120 02	9.3020 02	9.2880 02	9.2710 02	9.2510 02	9.2300 02	9.2070 02	9.1850 02	9.1630 02
59.0	9.3690 02	9.3660 02	9.3590 02	9.3480 02	9.3320 02	9.3130 02	9.2900 02	9.2660 02	9.2390 02	9.2130 02	9.1870 02
58.0	9.4210 02	9.4180 02	9.4100 02	9.3970 02	9.3800 02	9.3580 02	9.3330 02	9.3050 02	9.2750 02	9.2450 02	9.2150 02
57.0	9.4770 02	9.4740 02	9.4660 02	9.4510 02	9.4320 02	9.4070 02	9.3790 02	9.3480 02	9.3140 02	9.2800 02	9.2460 02
56.0	9.5390 02	9.5350 02	9.5260 02	9.5100 02	9.4880 02	9.4610 02	9.4300 02	9.3950 02	9.3570 02	9.3180 02	9.2790 02
55.0	9.6050 02	9.6010 02	9.5910 02	9.5730 02	9.5490 02	9.5200 02	9.4850 02	9.4460 02	9.4040 02	9.3600 02	9.3160 02
54.0	9.6760 02	9.6720 02	9.6600 02	9.6410 02	9.6150 02	9.5830 02	9.5440 02	9.5010 02	9.4550 02	9.4060 02	9.3570 02
53.0	9.7520 02	9.7480 02	9.7360 02	9.7150 02	9.6860 02	9.6510 02	9.6090 02	9.5610 02	9.5100 02	9.4560 02	9.4010 02
52.0	9.8350 02	9.8300 02	9.8160 02	9.7940 02	9.7630 02	9.7240 02	9.6780 02	9.6260 02	9.5700 02	9.5100 02	9.4490 02
51.0	9.9230 02	9.9180 02	9.9030 02	9.8780 02	9.8450 02	9.8020 02	9.7530 02	9.6960 02	9.6340 02	9.5690 02	9.5020 02
50.0	1.0020 03	1.0010 03	9.9960 02	9.9690 02	9.9330 02	9.8870 02	9.8330 02	9.7710 02	9.7040 02	9.6320 02	9.5580 02
49.0	1.0120 03	1.0110 03	1.0090 03	1.0070 03	1.0030 03	9.9770 02	9.9190 02	9.8520 02	9.7790 02	9.7000 02	9.6190 02
48.0	1.0230 03	1.0220 03	1.0200 03	1.0170 03	1.0130 03	1.0070 03	1.0010 03	9.9390 02	9.8590 02	9.7740 02	9.6850 02
47.0	1.0340 03	1.0330 03	1.0310 03	1.0280 03	1.0240 03	1.0180 03	1.0110 03	1.0030 03	9.9450 02	9.8530 02	9.7570 02
46.0	1.0460 03	1.0460 03	1.0430 03	1.0400 03	1.0350 03	1.0290 03	1.0220 03	1.0130 03	1.0040 03	9.9380 02	9.8340 02
45.0	1.0590 03	1.0590 03	1.0560 03	1.0530 03	1.0470 03	1.0410 03	1.0330 03	1.0240 03	1.0140 03	1.0030 03	9.9160 02
44.0	1.0730 03	1.0730 03	1.0700 03	1.0660 03	1.0600 03	1.0530 03	1.0450 03	1.0350 03	1.0240 03	1.0130 03	1.0010 03
43.0	1.0880 03	1.0870 03	1.0850 03	1.0800 03	1.0750 03	1.0670 03	1.0580 03	1.0480 03	1.0360 03	1.0230 03	1.0100 03
42.0	1.1040 03	1.1030 03	1.1000 03	1.0960 03	1.0900 03	1.0810 03	1.0720 03	1.0610 03	1.0480 03	1.0350 03	1.0200 03
41.0	1.1210 03	1.1200 03	1.1170 03	1.1120 03	1.1060 03	1.0970 03	1.0870 03	1.0750 03	1.0610 03	1.0470 03	1.0310 03
40.0	1.1390 03	1.1380 03	1.1350 03	1.1300 03	1.1230 03	1.1130 03	1.1020 03	1.0900 03	1.0750 03	1.0600 03	1.0430 03
39.0	1.1590 03	1.1570 03	1.1540 03	1.1490 03	1.1410 03	1.1310 03	1.1190 03	1.1060 03	1.0910 03	1.0740 03	1.0560 03
38.0	1.1790 03	1.1780 03	1.1740 03	1.1690 03	1.1600 03	1.1500 03	1.1380 03	1.1230 03	1.1070 03	1.0890 03	1.0700 03
37.0	1.2010 03	1.2000 03	1.1960 03	1.1900 03	1.1810 03	1.1700 03	1.1570 03	1.1420 03	1.1240 03	1.1050 03	1.0850 03
36.0	1.2250 03	1.2230 03	1.2190 03	1.2130 03	1.2040 03	1.1920 03	1.1780 03	1.1620 03	1.1430 03	1.1230 03	1.1010 03
35.0	1.2500 03	1.2490 03	1.2440 03	1.2370 03	1.2280 03	1.2150 03	1.2000 03	1.1830 03	1.1630 03	1.1410 03	1.1180 03
34.0	1.2770 03	1.2760 03	1.2710 03	1.2640 03	1.2530 03	1.2400 03	1.2240 03	1.2060 03	1.1850 03	1.1620 03	1.1370 03
33.0	1.3060 03	1.3040 03	1.3000 03	1.2920 03	1.2810 03	1.2670 03	1.2500 03	1.2310 03	1.2080 03	1.1830 03	1.1570 03
32.0	1.3370 03	1.3360 03	1.3310 03	1.3220 03	1.3110 03	1.2960 03	1.2780 03	1.2570 03	1.2330 03	1.2070 03	1.1780 03
31.0	1.3710 03	1.3690 03	1.3640 03	1.3550 03	1.3430 03	1.3270 03	1.3080 03	1.2860 03	1.2610 03	1.2320 03	1.2020 03
30.0	1.4070 03	1.4050 03	1.3990 03	1.3900 03	1.3770 03	1.3610 03	1.3400 03	1.3170 03	1.2900 03	1.2600 03	1.2270 03
29.0	1.4460 03	1.4440 03	1.4380 03	1.4280 03	1.4140 03	1.3970 03	1.3750 03	1.3500 03	1.3220 03	1.2900 03	1.2550 03
28.0	1.4880 03	1.4860 03	1.4790 03	1.4690 03	1.4550 03	1.4360 03	1.4130 03	1.3870 03	1.3560 03	1.3220 03	1.2850 03
27.0	1.5330 03	1.5310 03	1.5240 03	1.5130 03	1.4980 03	1.4780 03	1.4540 03	1.4260 03	1.3940 03	1.3570 03	1.3180 03
26.0	1.5830 03	1.5800 03	1.5730 03	1.5620 03	1.5450 03	1.5250 03	1.4990 03	1.4690 03	1.4340 03	1.3960 03	1.3530 03
25.0	1.6360 03	1.6340 03	1.6270 03	1.6140 03	1.5970 03	1.5750 03	1.5480 03	1.5160 03	1.4790 03	1.4380 03	1.3930 03
24.0	1.6950 03	1.6930 03	1.6850 03	1.6720 03	1.6530 03	1.6300 03	1.6010 03	1.5670 03	1.5280 03	1.4840 03	1.4350 03
23.0	1.7590 03	1.7570 03	1.7480 03	1.7340 03	1.7150 03	1.6900 03	1.6590 03	1.6230 03	1.5810 03	1.5340 03	1.4830 03
22.0	1.8300 03	1.8270 03	1.8180 03	1.8030 03	1.7830 03	1.7560 03	1.7230 03	1.6850 03	1.6400 03	1.5900 03	1.5350 03
21.0	1.9070 03	1.9040 03	1.8950 03	1.8790 03	1.8570 03	1.8290 03	1.7940 03	1.7530 03	1.7050 03	1.6520 03	1.5920 03
20.0	1.9930 03	1.9900 03	1.9800 03	1.9630 03	1.9400 03	1.9100 03	1.8730 03	1.8290 03	1.7780 03	1.7200 03	1.6570 03
19.0	2.0890 03	2.0850 03	2.0750 03	2.0570 03	2.0320 03	1.9990 03	1.9600 03	1.9130 03	1.8580 03	1.7970 03	1.7280 03
18.0	2.1950 03	2.1920 03	2.1800 03	2.1610 03	2.1340 03	2.1000 03	2.0570 03	2.0070 03	1.9480 03	1.8820 03	1.8090 03
17.0	2.3150 03	2.3110 03	2.2990 03	2.2780 03	2.2500 03	2.2130 03	2.1670 03	2.1130 03	2.0500 03	1.9790 03	1.8990 03
16.0	2.4500 03	2.4460 03	2.4330 03	2.4110 03	2.3800 03	2.3400 03	2.2910 03	2.2330 03	2.1650 03	2.0880 03	2.0020 03
15.0	2.6040 03	2.6000 03	2.5850 03	2.5620 03	2.5280 03	2.4850 03	2.4320 03	2.3690 03	2.2960 03	2.2130 03	2.1200 03
14.0	2.7810 03	2.7760 03	2.7600 03	2.7350 03	2.6990 03	2.6520 03	2.5950 03	2.5260 03	2.4470 03	2.3560 03	2.2550 03
13.0	2.9850 03	2.9800 03	2.9630 03	2.9350 03	2.8960 03	2.8450 03	2.7830 03	2.7080 03	2.6220 03	2.5230 03	2.4120 03
12.0	3.2250 03	3.2190 03	3.2000 03	3.1700 03	3.1270 03	3.0720 03	3.0030 03	2.9220 03	2.8270 03	2.7180 03	2.5970 03
11.0	3.5080 03	3.5020 03	3.4820 03	3.4480 03	3.4010 03	3.3400 03	3.2650 03	3.1750 03	3.0700 03	2.9510 03	2.8170 03
10.0	3.8490 03	3.8420 03	3.8200 03	3.7830 03	3.7310 03	3.6630 03	3.5800 03	3.4800 03	3.3640 03	3.2310 03	3.0820 03
9.0	4.2680 03	4.2590 03	4.2350 03	4.1930 03	4.1350 03	4.0590 03	3.9660 03	3.8540 03	3.7240 03	3.5750 03	3.4080 03
8.0	4.7910 03	4.7820 03	4.7540 03	4.7070 03	4.6410 03	4.5550 03	4.4500 03	4.3230 03	4.1760 03	4.0070 03	3.8170 03
7.0	5.4660 03	5.4550 03	5.4230 03	5.3690 03	5.2940 03	5.1950 03	5.0730 03	4.9280 03	4.7580 03	4.5640 03	4.3450 03
6.0	6.3670 03	6.3550 03	6.3170 03	6.2540 03	6.1650 03	6.0500 03	5.9070 03	5.7360 03	5.5370 03	5.3090 03	5.0520 03
5.0	7.6310 03	7.6160 03	7.5700 03	7.4940 03	7.3870 03	7.2480 03	7.0760 03	6.8710 03	6.6310 03	6.3560 03	6.0450 03
4.0	9.5280 03	9.5090 03	9.4520 03	9.3570 03	9.2230 03	9.0480 03	8.8330 03	8.5750 03	8.2740 03	7.9280 03	7.5380 03
3.0	1.2690 04	1.2670 04	1.2590 04	1.2460 04	1.2280 04	1.2050 04	1.1760 04	1.1420 04	1.1020 04	1.0550 04	1.0030 04
2.0	1.9030 04	1.8990 04	1.8870 04	1.8680 04	1.8410 04	1.8060 04	1.7630 04	1.7110 04	1.6510 04	1.5810 04	1.5030 04
1.0	3.8020 04	3.7940 04	3.7710 04	3.7330 04	3.6790 04	3.6090 04	3.5230 04	3.4190 04	3.2980 04	3.1590 04	3.0020 04
0.0	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06	1.0000 06

Fig. 2. RMS position errors for all equatorial satellites. (All values in meters.)

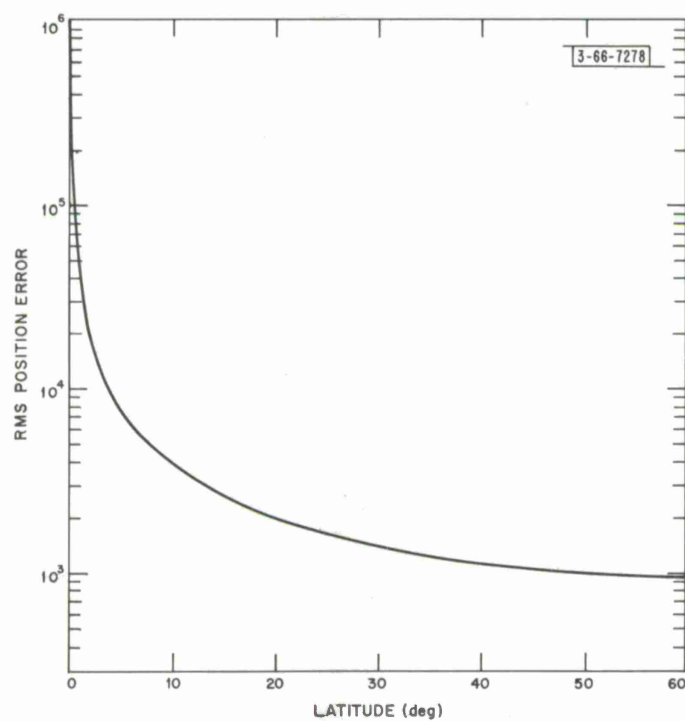


Fig. 3. Cross section of RMS position error vs latitude (longitude = 0°).

3-66-7279

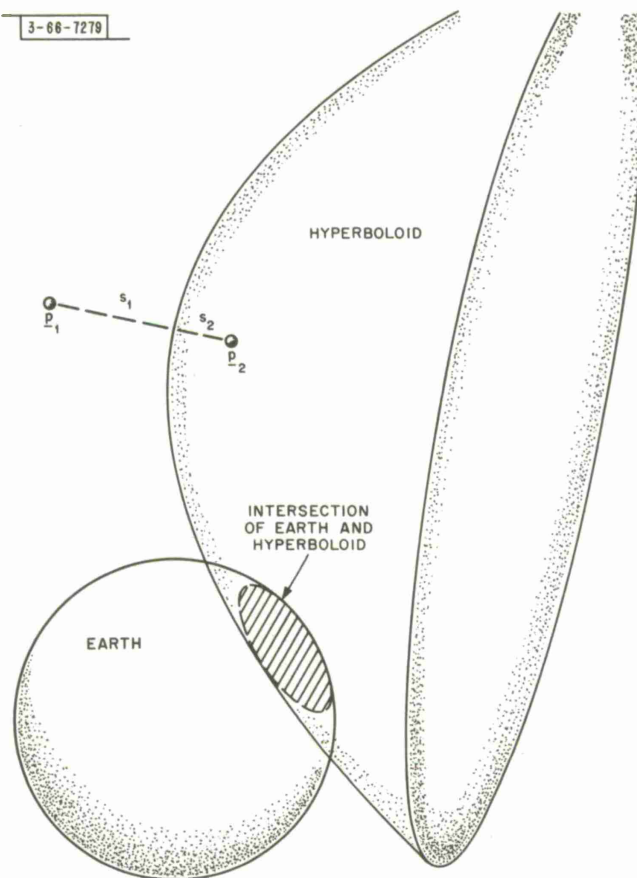
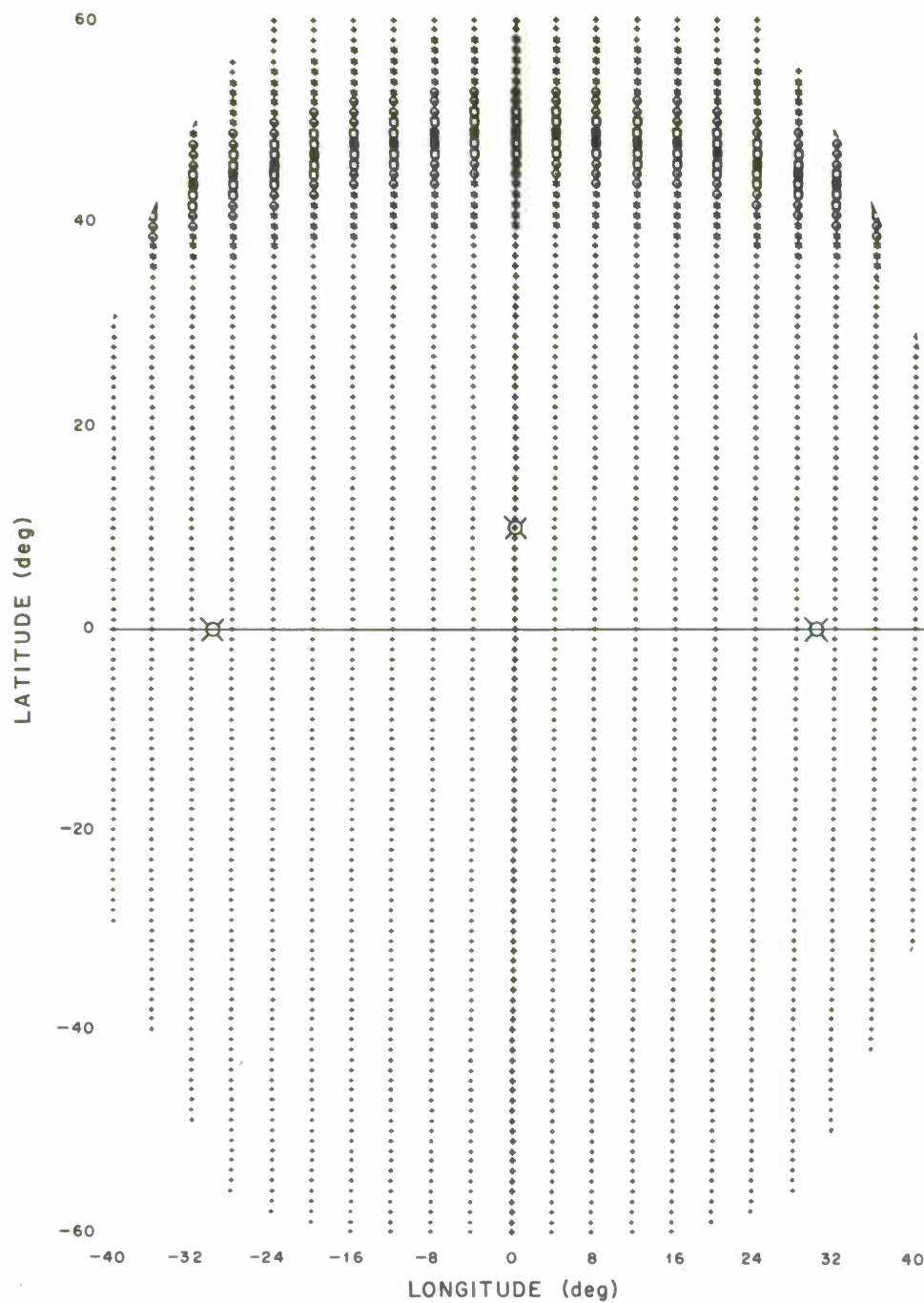


Fig. 4. Intersection of hyperboloid with tangent plane of earth.

USER POSITION															
LAT	LONG	0.0	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00			
60.00	2.110E 00	2.107E 00	2.101E 00	2.091E 00	2.076E 00	2.059E 00	2.038E 00	2.015E 00	1.990E 00	1.964E 00	1.937E 00	1.937E 00			
59.00	2.124E 00	2.121E 00	2.114E 00	2.103E 00	2.088E 00	2.068E 00	2.046E 00	2.021E 00	1.994E 00	1.966E 00	1.937E 00	1.937E 00			
58.00	2.138E 00	2.136E 00	2.128E 00	2.116E 00	2.100E 00	2.079E 00	2.055E 00	2.028E 00	1.999E 00	1.968E 00	1.937E 00	1.937E 00			
57.00	2.154E 00	2.152E 00	2.144E 00	2.130E 00	2.112E 00	2.090E 00	2.064E 00	2.035E 00	2.004E 00	1.971E 00	1.937E 00	1.937E 00			
56.00	2.171E 00	2.168E 00	2.160E 00	2.145E 00	2.126E 00	2.102E 00	2.074E 00	2.043E 00	2.009E 00	1.974E 00	1.938E 00	1.938E 00			
55.00	2.189E 00	2.186E 00	2.177E 00	2.161E 00	2.141E 00	2.115E 00	2.085E 00	2.051E 00	2.015E 00	1.977E 00	1.939E 00	1.939E 00			
54.00	2.208E 00	2.205E 00	2.195E 00	2.178E 00	2.156E 00	2.129E 00	2.096E 00	2.060E 00	2.022E 00	1.981E 00	1.940E 00	1.940E 00			
53.00	2.228E 00	2.225E 00	2.214E 00	2.196E 00	2.173E 00	2.143E 00	2.109E 00	2.070E 00	2.029E 00	1.985E 00	1.941E 00	1.941E 00			
52.00	2.250E 00	2.246E 00	2.234E 00	2.216E 00	2.190E 00	2.159E 00	2.122E 00	2.081E 00	2.036E 00	1.990E 00	1.943E 00	1.943E 00			
51.00	2.272E 00	2.268E 00	2.256E 00	2.236E 00	2.209E 00	2.175E 00	2.136E 00	2.092E 00	2.045E 00	1.995E 00	1.945E 00	1.945E 00			
50.00	2.296E 00	2.292E 00	2.279E 00	2.258E 00	2.229E 00	2.193E 00	2.151E 00	2.104E 00	2.053E 00	2.001E 00	1.947E 00	1.947E 00			
49.00	2.322E 00	2.317E 00	2.303E 00	2.281E 00	2.250E 00	2.211E 00	2.167E 00	2.117E 00	2.063E 00	2.007E 00	1.950E 00	1.950E 00			
48.00	2.349E 00	2.344E 00	2.329E 00	2.305E 00	2.272E 00	2.231E 00	2.184E 00	2.131E 00	2.074E 00	2.014E 00	1.953E 00	1.953E 00			
47.00	2.378E 00	2.372E 00	2.357E 00	2.331E 00	2.296E 00	2.252E 00	2.202E 00	2.146E 00	2.085E 00	2.022E 00	1.957E 00	1.957E 00			
46.00	2.408E 00	2.402E 00	2.386E 00	2.358E 00	2.321E 00	2.275E 00	2.221E 00	2.161E 00	2.097E 00	2.030E 00	1.961E 00	1.961E 00			
45.00	2.440E 00	2.434E 00	2.417E 00	2.388E 00	2.348E 00	2.299E 00	2.242E 00	2.178E 00	2.110E 00	2.039E 00	1.966E 00	1.966E 00			
44.00	2.475E 00	2.468E 00	2.449E 00	2.419E 00	2.377E 00	2.325E 00	2.264E 00	2.197E 00	2.124E 00	2.049E 00	1.972E 00	1.972E 00			
43.00	2.511E 00	2.504E 00	2.484E 00	2.451E 00	2.407E 00	2.352E 00	2.288E 00	2.216E 00	2.140E 00	2.059E 00	1.978E 00	1.978E 00			
42.00	2.549E 00	2.542E 00	2.521E 00	2.486E 00	2.439E 00	2.381E 00	2.313E 00	2.237E 00	2.156E 00	2.071E 00	1.985E 00	1.985E 00			
41.00	2.590E 00	2.583E 00	2.560E 00	2.524E 00	2.474E 00	2.412E 00	2.340E 00	2.260E 00	2.174E 00	2.084E 00	1.993E 00	1.993E 00			
40.00	2.634E 00	2.626E 00	2.602E 00	2.563E 00	2.510E 00	2.445E 00	2.369E 00	2.284E 00	2.193E 00	2.098E 00	2.002E 00	2.002E 00			
39.00	2.680E 00	2.672E 00	2.647E 00	2.605E 00	2.549E 00	2.480E 00	2.400E 00	2.310E 00	2.214E 00	2.113E 00	2.012E 00	2.012E 00			
38.00	2.729E 00	2.721E 00	2.694E 00	2.650E 00	2.591E 00	2.518E 00	2.433E 00	2.338E 00	2.236E 00	2.130E 00	2.023E 00	2.023E 00			
37.00	2.782E 00	2.773E 00	2.744E 00	2.698E 00	2.636E 00	2.558E 00	2.468E 00	2.368E 00	2.260E 00	2.148E 00	2.035E 00	2.035E 00			
36.00	2.838E 00	2.828E 00	2.798E 00	2.750E 00	2.683E 00	2.601E 00	2.506E 00	2.400E 00	2.286E 00	2.168E 00	2.048E 00	2.048E 00			
35.00	2.898E 00	2.887E 00	2.856E 00	2.804E 00	2.734E 00	2.648E 00	2.547E 00	2.435E 00	2.314E 00	2.189E 00	2.063E 00	2.063E 00			
34.00	2.962E 00	2.951E 00	2.917E 00	2.863E 00	2.789E 00	2.697E 00	2.591E 00	2.472E 00	2.345E 00	2.213E 00	2.079E 00	2.079E 00			
33.00	3.030E 00	3.019E 00	2.983E 00	2.926E 00	2.848E 00	2.751E 00	2.638E 00	2.513E 00	2.378E 00	2.238E 00	2.097E 00	2.097E 00			
32.00	3.104E 00	3.091E 00	3.054E 00	2.993E 00	2.911E 00	2.808E 00	2.689E 00	2.557E 00	2.414E 00	2.266E 00	2.117E 00	2.117E 00			
31.00	3.183E 00	3.170E 00	3.130E 00	3.066E 00	2.978E 00	2.870E 00	2.744E 00	2.604E 00	2.453E 00	2.297E 00	2.139E 00	2.139E 00			
30.00	3.268E 00	3.254E 00	3.212E 00	3.144E 00	3.051E 00	2.937E 00	2.804E 00	2.655E 00	2.496E 00	2.331E 00	2.164E 00	2.164E 00			
29.00	3.359E 00	3.344E 00	3.300E 00	3.228E 00	3.130E 00	3.009E 00	2.868E 00	2.711E 00	2.543E 00	2.368E 00	2.191E 00	2.191E 00			
28.00	3.458E 00	3.442E 00	3.395E 00	3.319E 00	3.216E 00	3.087E 00	2.938E 00	2.772E 00	2.594E 00	2.408E 00	2.221E 00	2.221E 00			
27.00	3.564E 00	3.548E 00	3.498E 00	3.418E 00	3.308E 00	3.172E 00	3.014E 00	2.838E 00	2.649E 00	2.453E 00	2.255E 00	2.255E 00			
26.00	3.680E 00	3.662E 00	3.610E 00	3.525E 00	3.409E 00	3.265E 00	3.097E 00	2.911E 00	2.710E 00	2.502E 00	2.292E 00	2.292E 00			
25.00	3.806E 00	3.787E 00	3.732E 00	3.641E 00	3.518E 00	3.365E 00	3.188E 00	2.990E 00	2.777E 00	2.557E 00	2.334E 00	2.334E 00			
24.00	3.943E 00	3.923E 00	3.865E 00	3.768E 00	3.638E 00	3.476E 00	3.287E 00	3.077E 00	2.852E 00	2.617E 00	2.380E 00	2.380E 00			
23.00	4.093E 00	4.072E 00	4.010E 00	3.908E 00	3.769E 00	3.597E 00	3.396E 00	3.173E 00	2.933E 00	2.684E 00	2.432E 00	2.432E 00			
22.00	4.258E 00	4.235E 00	4.169E 00	4.060E 00	3.913E 00	3.730E 00	3.517E 00	3.279E 00	3.024E 00	2.759E 00	2.490E 00	2.490E 00			
21.00	4.439E 00	4.415E 00	4.345E 00	4.229E 00	4.072E 00	3.877E 00	3.650E 00	3.397E 00	3.125E 00	2.842E 00	2.556E 00	2.556E 00			
20.00	4.639E 00	4.614E 00	4.539E 00	4.416E 00	4.248E 00	4.040E 00	3.798E 00	3.528E 00	3.238E 00	2.935E 00	2.630E 00	2.630E 00			
19.00	4.862E 00	4.835E 00	4.754E 00	4.623E 00	4.444E 00	4.222E 00	3.963E 00	3.674E 00	3.364E 00	3.041E 00	2.714E 00	2.714E 00			
18.00	5.110E 00	5.081E 00	4.995E 00	4.855E 00	4.663E 00	4.425E 00	4.148E 00	3.839E 00	3.507E 00	3.160E 00	2.809E 00	2.809E 00			
17.00	5.389E 00	5.358E 00	5.266E 00	5.115E 00	4.909E 00	4.654E 00	4.357E 00	4.025E 00	3.668E 00	3.296E 00	2.918E 00	2.918E 00			
16.00	5.704E 00	5.671E 00	5.572E 00	5.409E 00	5.188E 00	4.914E 00	4.594E 00	4.237E 00	3.852E 00	3.451E 00	3.044E 00	3.044E 00			
15.00	6.063E 00	6.027E 00	5.920E 00	5.745E 00	5.506E 00	5.210E 00	4.864E 00	4.478E 00	4.063E 00	3.629E 00	3.189E 00	3.189E 00			
14.00	6.474E 00	6.435E 00	6.319E 00	6.129E 00	5.871E 00	5.550E 00	5.176E 00	4.758E 00	4.307E 00	3.836E 00	3.358E 00	3.358E 00			
13.00	6.950E 00	6.908E 00	6.782E 00	6.575E 00	6.294E 00	5.945E 00	5.537E 00	5.082E 00	4.591E 00	4.078E 00	3.557E 00	3.557E 00			
12.00	7.508E 00	7.461E 00	7.323E 00	7.098E 00	6.790E 00	6.408E 00	5.962E 00	5.464E 00	4.927E 00	4.365E 00	3.793E 00	3.793E 00			
11.00	8.168E 00	8.117E 00	7.965E 00	7.717E 00	7.378E 00	6.958E 00	6.467E 00	5.919E 00	5.327E 00	4.708E 00	4.077E 00	4.077E 00			
10.00	8.963E 00	8.906E 00	8.738E 00	8.463E 00	8.087E 00	7.621E 00	7.077E 00	6.469E 00	5.812E 00	5.123E 00	4.422E 00	4.422E 00			
9.00	9.936E 00	9.873E 00	9.685E 00	9.377E 00	8.956E 00	8.435E 00	7.826E 00	7.145E 00	6.409E 00	5.637E 00	4.851E 00	4.851E 00			
8.00	1.116E 01	1.108E 01	1.087E 01	1.052E 01	1.005E 01	9.456E 00	8.767E 00	7.994E 00	7.161E 00	6.286E 00	5.393E 00	5.393E 00			
7.00	1.273E 01	1.265E 01	1.240E 01	1.200E 01	1.145E 01	1.077E 01	9.980E 00	9.093E 00	8.134E 00	7.127E 00	6.099E 00	6.099E 00			
6.00	1.483E 01	1.473E 01	1.444E 01	1.397E 01	1.333E 01	1.254E 01	1.160E 01	1.056E 01	9.439E 00	8.257E 00	7.049E 00	7.049E 00			
5.00	1.777E 01	1.765E 01	1.731E 01	1.674E 01	1.597E 01	1.501E 01	1.389E 01	1.263E 01	1.127E 01	9.849E 00	8.392E 00	8.392E 00			
4.00	2.219E 01	2.204E 01	2.161E 01	2.090E 01	1.993E 01	1.872E 01	1.732E 01	1.575E 01	1.404E 01	1.225E 01	1.042E 01	1.042E 01			
3.00	2.956E 01	2.938E 01	2.879E 01	2.784E 01	2.654E 01	2.493E 01	2.306E 01	2.095E 01	1.866E 01	1.628E 01	1.382E 01	1.382E 01			
2.00	4.431E 01	4.402E 01	4.316E 01	4.173E 01	3.979E 01	3.737E 01	3.453E 01	3.138E 01	2.794E 01	2.433E 01	2.067E 01	2.067E 01			
1.00	8.875E 01	8.838E 01	8.628E 01	8.368E 01	7.963E 01	7.488E 01	6.916E 01	6.276E 01	5.589E 01	4.863E 01	4.125E 01	4.125E 01			
0.0	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE	ZERO ANGLE			



VALUES OF Q IN THE RANGE

- + < 60
- * 60 - 69
- ⊙ 70 - 79
- 80 - 100
- ⊠ > 100 AND $\theta = 0^\circ$

$$\text{WHERE } Q = \frac{100}{3} \log \left(\frac{1}{\sin \theta} \right) + 33$$

-66-7281

Fig. 6(a). $1/\sin \theta$ for an east-west symmetric satellite geometry.

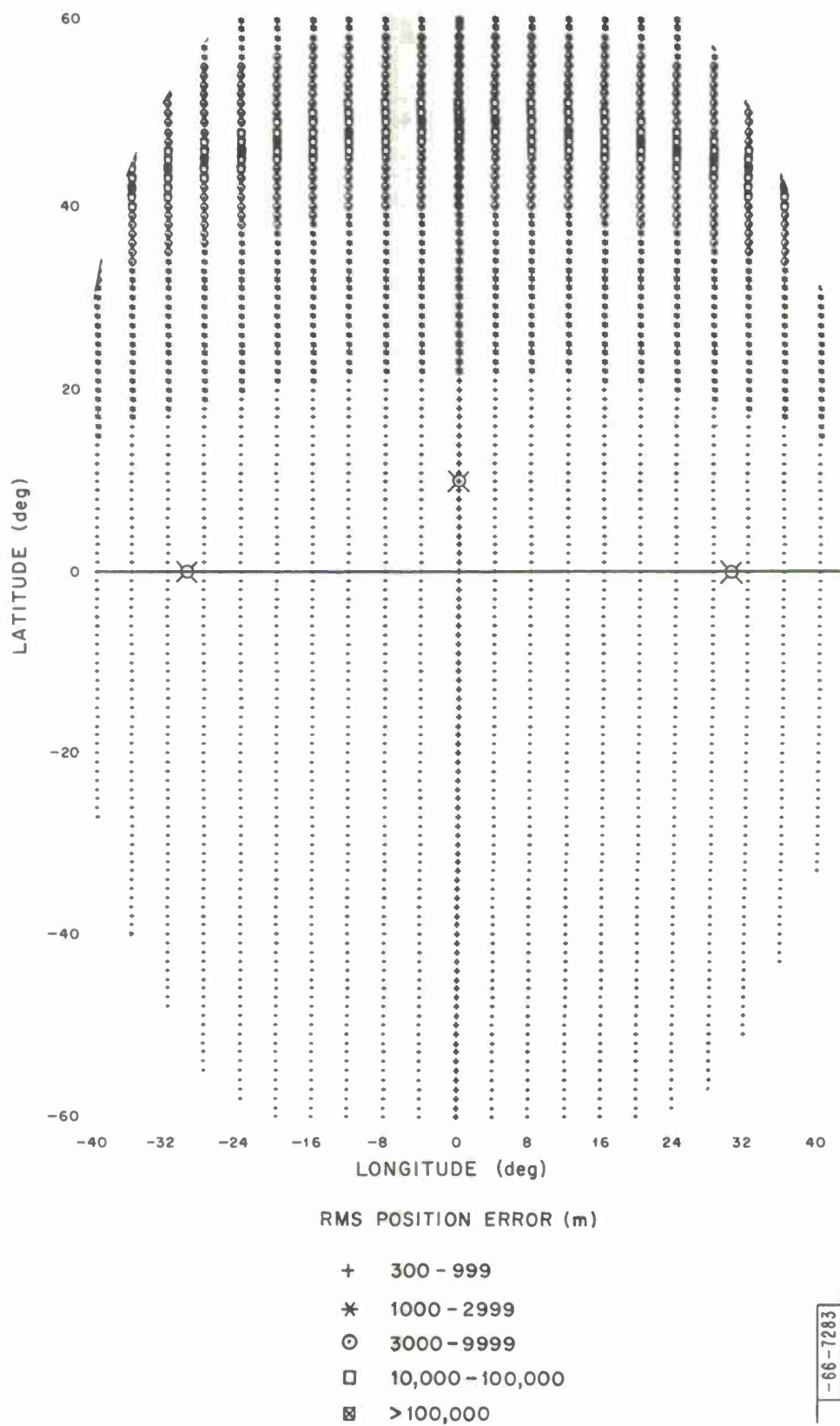
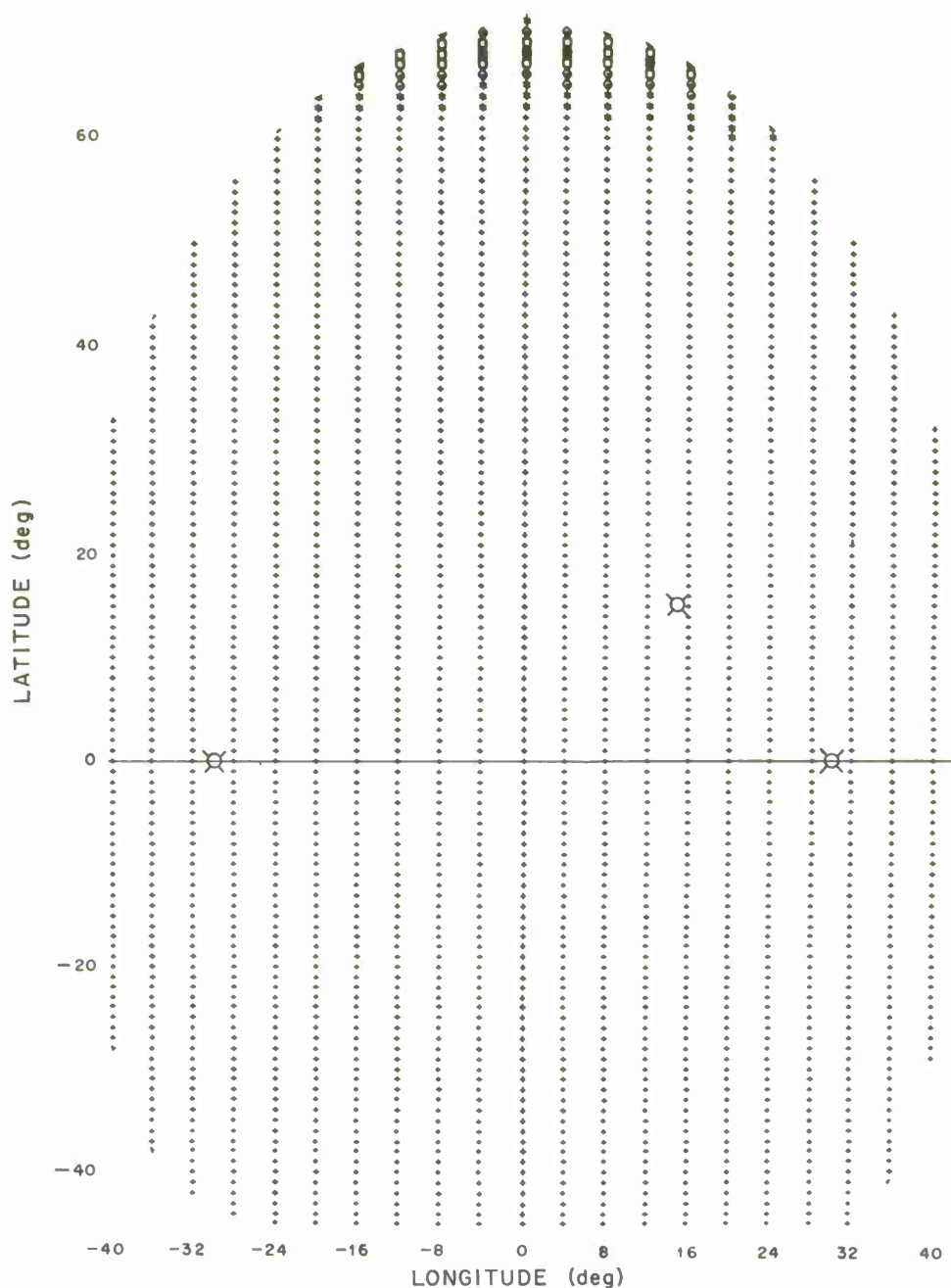


Fig. 6(b). RMS position errors for an east-west symmetric satellite geometry.



VALUES OF Q IN THE RANGE

- + < 60
- * 60 - 69
- ⊗ 70 - 79
- 80 - 100
- ▣ > 100 AND $\theta = 0^\circ$

WHERE $Q = \frac{100}{3} \log \left(\frac{1}{\sin \theta} \right) + 33$

-66-7284

Fig. 7(a). $1/\sin \theta$ for an unsymmetric satellite geometry.

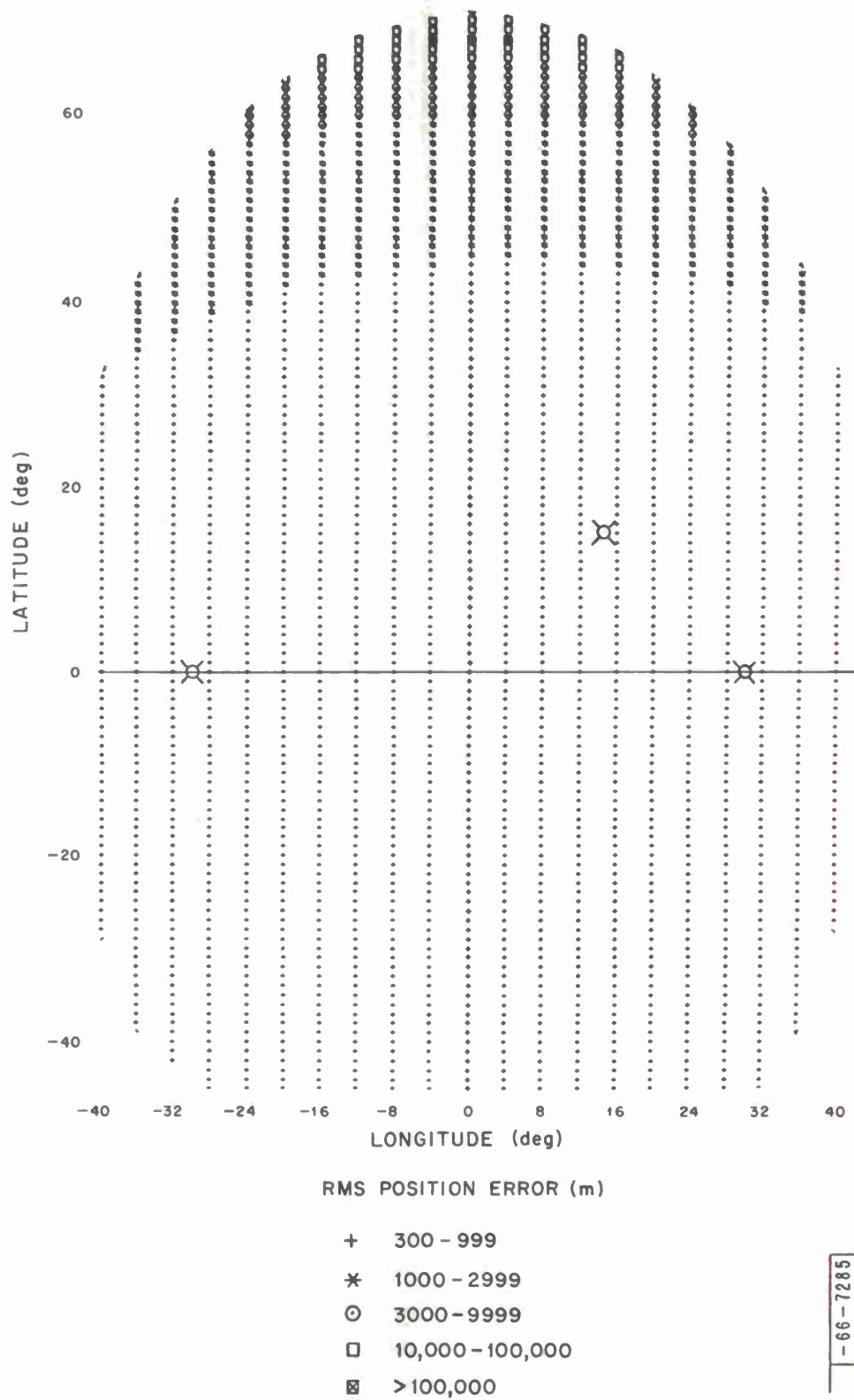


Fig. 7(b). RMS position errors for an unsymmetric satellite geometry.

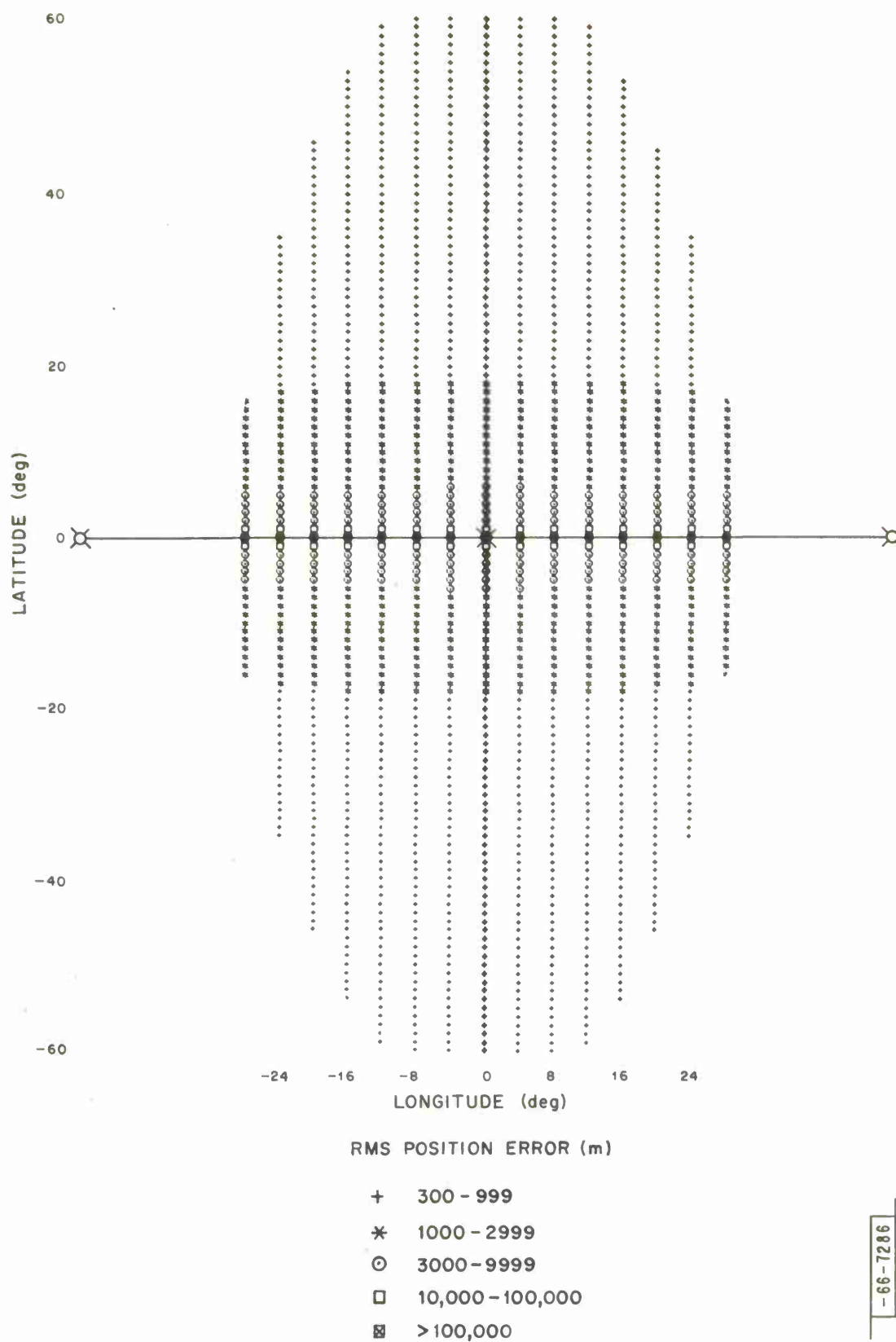


Fig. 8(a). RMS position errors for satellites at 45-0, 0-0, -45-0.

-66-7286

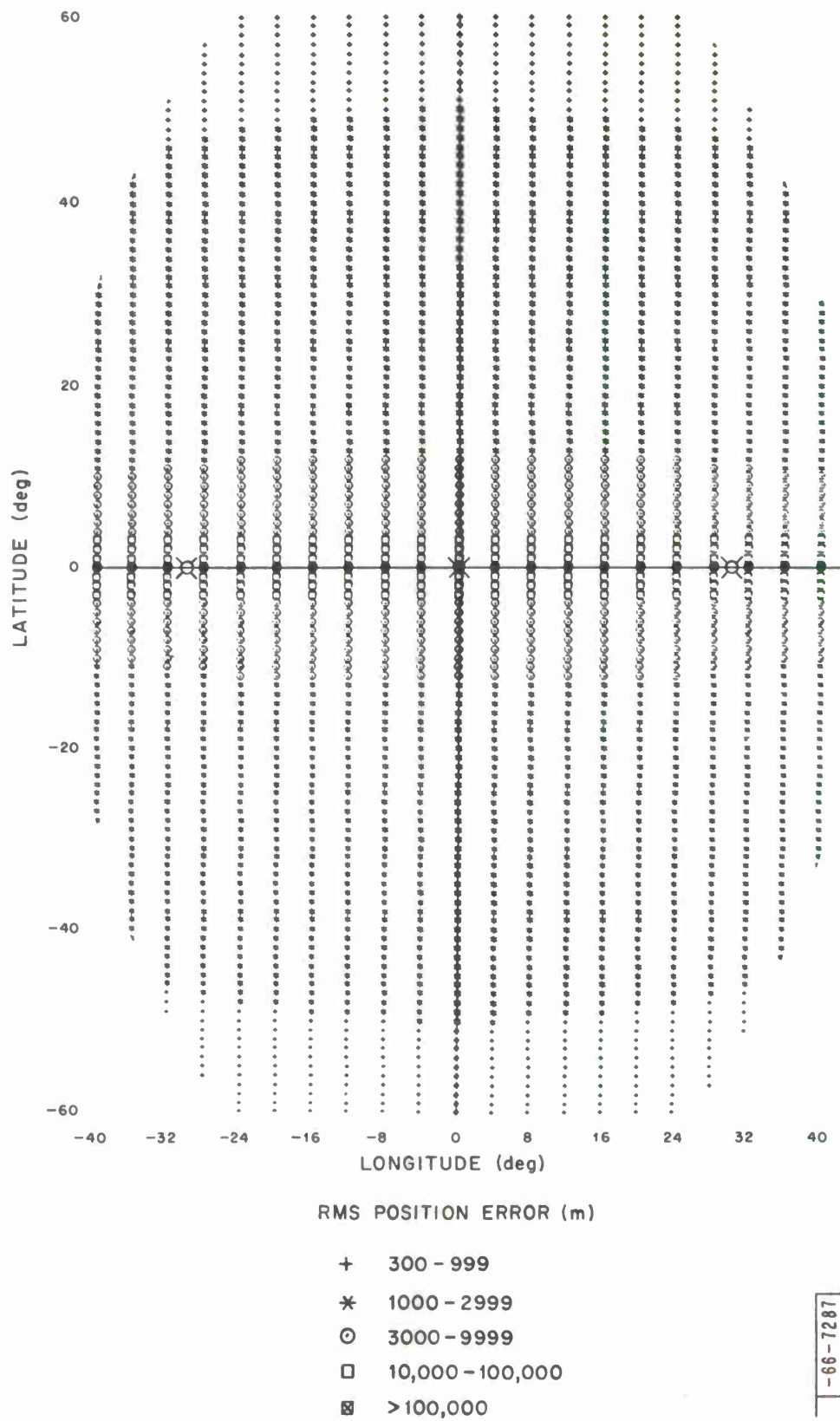


Fig. 8(b). RMS position errors for satellites at 30-0, 0-0, -30-0.

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Lincoln Laboratory, M. I. T.		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP None	
3. REPORT TITLE Navigation with High-Altitude Satellites: A Study of the Effects of Satellite-User Geometry on Position Accuracy			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Note			
5. AUTHOR(S) (Last name, first name, initial) Sullivan, Carole D.			
6. REPORT DATE 24 February 1967		7a. TOTAL NO. OF PAGES 24	7b. NO. OF REFS 2
8a. CONTRACT OR GRANT NO. AF 19(628)-5167		9a. ORIGINATOR'S REPORT NUMBER(S) Technical Note 1967-18	
b. PROJECT NO. 649L		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) ESD-TR-67-168	
c.			
d.			
10. AVAILABILITY/LIMITATION NOTICES Distribution of this document is unlimited.			
11. SUPPLEMENTARY NOTES None		12. SPONSORING MILITARY ACTIVITY Air Force Systems Command, USAF	
13. ABSTRACT <p>An error analysis of an hyperbolic navigation system using high-altitude satellites revealed that certain satellite arrangements result in long, narrow corridors on the earth (singular regions), in which navigation errors are very large. The study showed that in these singular regions the satellite-user geometry results in navigation equations which are sensitive to measurement errors and thus cannot be solved accurately for all three user coordinates.</p>			
14. KEY WORDS hyperbolic navigation system singular region satellite-user geometry			

Printed by
United States Air Force
L. G. Hanscom Field
Bedford, Massachusetts

